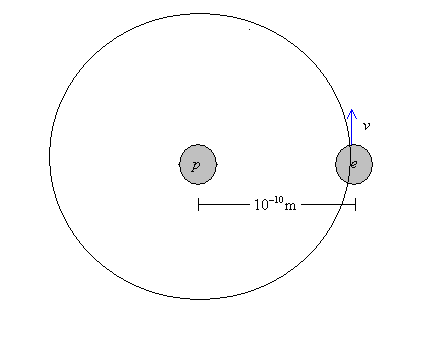
Magnetic Field Problems

**Problem**

Consider an H atom. In the ground state, the electron circles the nucleus with a speed of roughly v ≈ 1.6×106m/s. What is the magnetic field created by the electron at the nucleus?

**Solution**

We have:



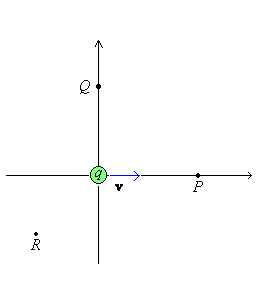
Using our formula above, the magnetic field is:



which is pretty large considering the Earth’s magnetic field is ~ 50μT, almost a million times weaker.

**Problem**

For the charge below (q = 5μC, and v = 3×106m/s) , what is the magnetic field and electric field at the point P = (5,0), Q = (0,5), R = (-5,-5)?



**Solution**

The electric field at point P is:



The magnetic field at point P is:



So the magnetic field at point P is zero. And this is because the angle between v and r is 0˚. Generally speaking then, the magnetic field is always zero right in front or behind the velocity of the charge. Let’s look at point Q,



The magnetic field at point Q is:



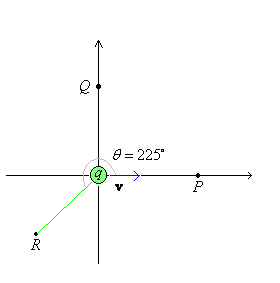
So the field at point Q is going out of the page, with a strength of 40nT. As you can see, magnetic field strengths are usually quite a bit smaller than electric field strengths. Now for point R,



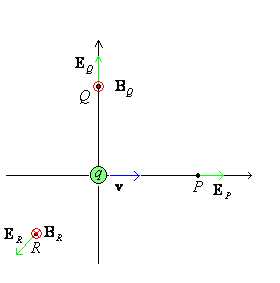
where **r** is a unit vector pointing away from the charge. The magnetic field at point R is:



The angle for point R is shown below:

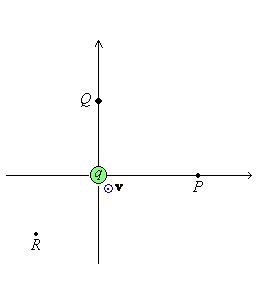


So we see that the field here is moving down into the page. So altogether the fields would be pointing in the following directions,



**Problem**

Consider a charge q = 3C at the origin moving upwards along the z-axis with speed v = 107m/s. What is the field produced at the coordinate P = (5,0), Q = (0,5) and R = (-5,-5)?



**Solution**

The circle with the dot indicates that the proton is moving upwards, out of the page. The field at point P can be determined from the equation:



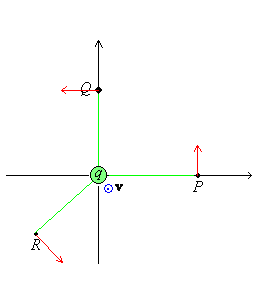
The field at Q is:



and the field at point R is:



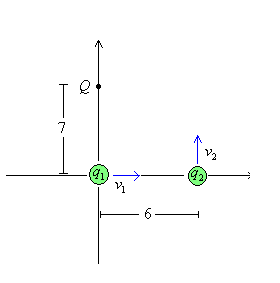
The fields are shown below in red, as well as the position vector **r** for those points. Observe that the the field is perpendicular to **v** and **r**, in accordance with the right hand rule for the cross-product q**v**×**r**.



If instead, q were -3C, then the field strengths would be the same, but the directions would be reversed.

**Problem**

Determine the net magnetic field created at the point Q by the following two charges: q1 = 5C traveling to the right with speed v1 = 40m/s, and q2 = -10C traveling north with speed v2 = 15m/s.



**Solution**

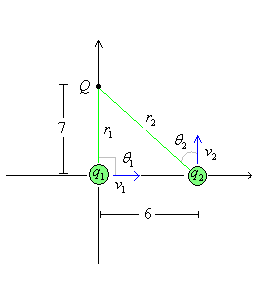
The field produced by q1 is:



and the field produced by the second charge would be:



The radii r1, r2 and angles θ1, θ2 are displayed below,



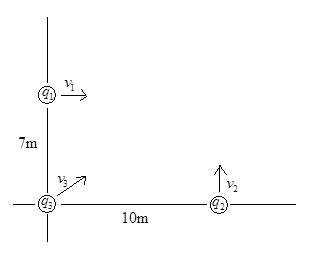
So the net field at point Q is:



So the net field has a strength of 225 nT and is coming out of the page.

**Problem**

Consider the following bunch of charges. Let q1 = 1μC, q2 =2μC, q3 = 3μC; v1 = 10m/s, v­2 = 20m/s, v3 = 30m/s, directed at 45◦ angle with respect to the horizontal. What is the magnitude of the net electric force that q1 and q2 exert on q3? What is the magnitude of the net magnetic force that q1 and q2 exert on q3?



**Solution**

The Electric field at the origin is:



and so the force is:



The magnetic field at the origin is:

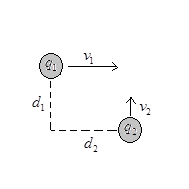


And so the magnitude of FB on q3 will be:



**Problem**

Two charges are moving perpendicularly to each other. Let q1 = 1C, q2 = 2C, v1 = 10m/s, v2 = 5m/s, d1 = 15cm, and d2 = 19cm. What is the magnitude and direction of the magnetic force that q1 exerts on q2? Note you’ll have to calculate the magnetic field that q1 sets up at the location of q2, and then calculate the force this field exerts on q2.



**Solution**

The field q1 sets up at q2’s position is:

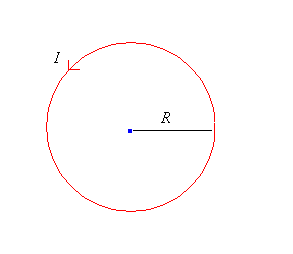


And so then the force on this charge is:



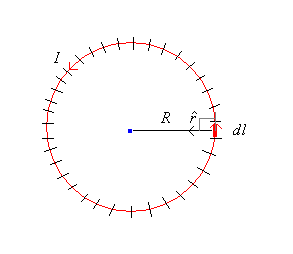
**Problem**

Let’s calculate **B** at center of circular wire radius R, current I.



**Solution**

To do so we break the wire up into little pieces, of which a typical one is shown marked in bold red.



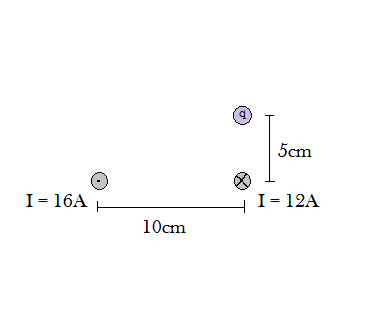
Performing the sum, over all the pieces in the circle,



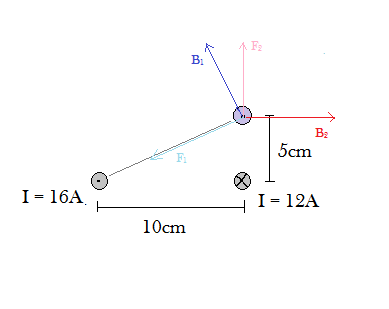
So we have,



**Question 7.** What is the magnitude and direction (CCW w/r to +x axis) of the force the two bottom infinite wires exert on the charge q = 6mC which is traveling out of the page with a velocity v = 1.3×106 m/s.



Fields and forces look like this:



And we have:



and,



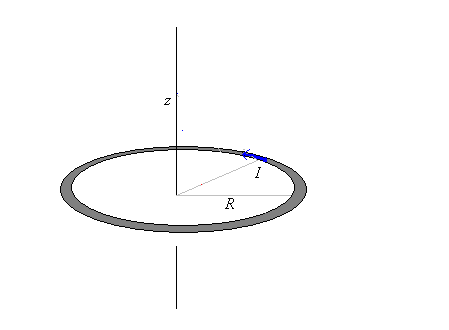
Adding the two together we get:



Note B1 = 28.6μT, and B2 = 48μT

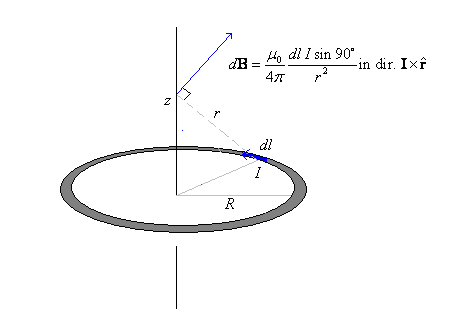
**Problem**

Consider the current loop again, radius R. Let’s calculate the field a point z above its center. So first we isolate a piece of wire dℓ, and draw the field **B**, which it creates. Note that the angle between **I** and **r** is 90 degrees.

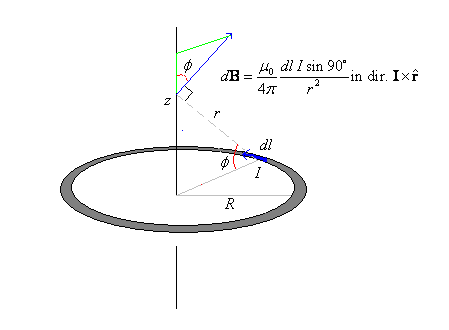


**Solution**

Here’s how it goes…

.

Now we need to break the field into its components. We’ll observe that the angle between d**B** and the z-axis is the same as the angle between the horizontal plane, and **r**.



Only the vertical component of d**B,** namely dBz = dBcosφ will survive the integration. And so,



Now



Therefore the field is:



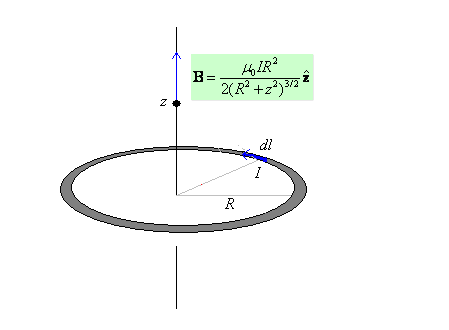
Observe that when z = 0, our formula reduces to the formula for B at the center of a wire.

**Problem**

According to Wikipedia, the Earth’s outer core is liquid, and circulating ionic currents in the outer core produce Earth’s magnetic field. The outer core has a radius of about 2000km from the center of the Earth. At the surface of the Earth, a distance of about 6400km from the center, the magnetic field has a strength of about 50μT. Given this, estimate the magnitude of the ionic currents creating the field.

**Solution**

Well, we know the field produced by a ring of current is:



So let R be the approximate radius of the outer core, and z be a point on the Earth’s surface, then solving for I we have,



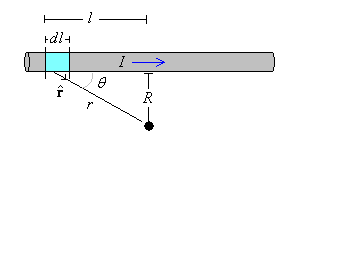
That’s 5 billion amps!

**Problem**

What is magnetic field due to an infinitely long straight wire carrying current I?

**Solution**

If the current is going clockwise, note that the direction would change. If we apply this equation to long straight wires, we do the same, breaking it up into little small pieces, of which the blue guy is typical.



From the formula, we have,



Evaluating the integral gives us 2/R2. Therefore, the answer is:



On the other side of the wire, the field would be pointing up, rather than down. In general, the magnetic field lines circulate around the wire, in direction which can be obtained using right-hand rule.



**Problem**

Lightning carries a current of about 50kA. What is the magnetic field produced by such a bolt, say 100m away?

**Solution**

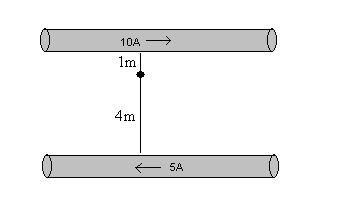
We can approximate the lightning bolt as a long straight wire. In that case,



The field would be about 10 times that of Earth’s natural magnetic field.

**Problem**

Let’s look at combinations of fields. For example,



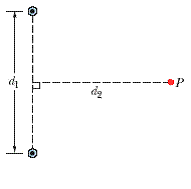
**Solution**

The **B**’s will add. So from the top (ask for directions first, then fill in the math, confirming)



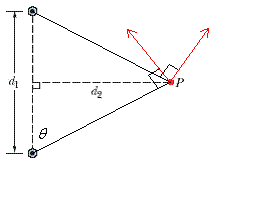
**Problem**

Figure below shows two very long straight wires (in cross section) that each carry a current of 5 A directly out of the page. Distance *d1* = 3m and distance *d2* = 5m. What is the magnitude of the net magnetic field at point *P*, which lies on a perpendicular bisector to the wires?



**Solution**

We have:



Angle θ that currents make with the point is:



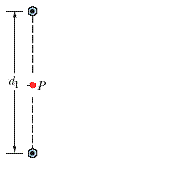
Magnetic field of each is:



Their horizontal components cancel, and their vertical components add, to give a net value of:



**Question 1.** Figure below shows two very long straight wires (in cross section). The top wire carries a current I1 = 5A, and the bottom wire carries a current I2 = 2A, each directed out of the page. Distance *d1* = 3m. What is the magnitude and direction of the net magnetic field at point *P*, directly inbetween the wires?



The field due to the top wire is:



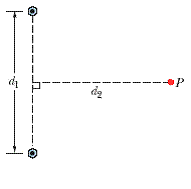
and the field due to the bottom wire is:



So the net field is B1 + B2 = 0.67μT – 0.267μT = 0.4 μT to the right.

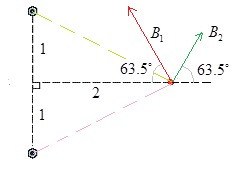
**Problem**

Consider the following situation. Two wires both carry current out of the page. The top wire carries a 5A current, and the bottom wire carries a current of 10A current. What is the magnitude and direction of the magnetic field at point P if d1 = d2 = 2m?



**Solution**

The magnetic fields look like this.



Adding the fields together we get:

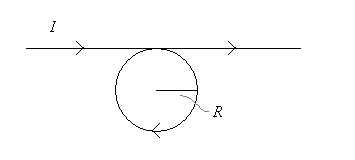


So the magnitude and direction of the field is:



**Problem**

A long straight wire is bent into the following configuration. What is the net magnetic field at the center of the loop? Let I = 10A, and R = 15cm.



**Solution**

The magnetic field is the sum of the fields due to the long straight wire and the circular loop. So we have,



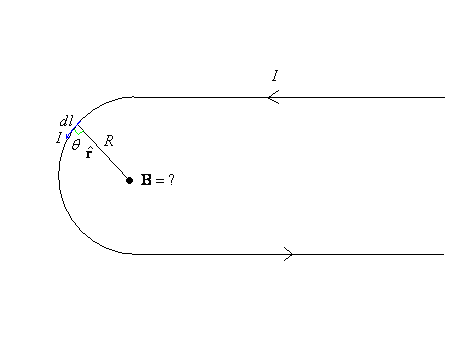
So we have,



**Problem**

A semi-circular wire is connected to two infinitely long wires, as shown below. Suppose that the current I = 1.2A, R = 15cm. Calculate the magnetic field at the indicated position. Give magnitude and direction. (Hint: break the wire up into three pieces → the semi-circular part and the two straight parts). You may find useful the fact:





**Solution**

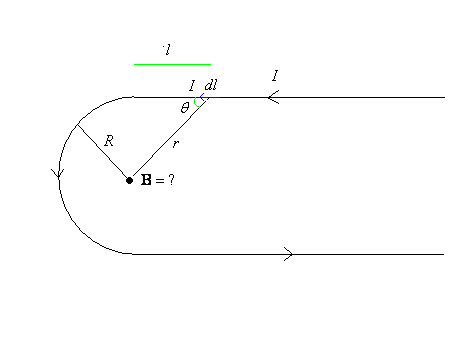
First, we’ll look at the field set up by the semi-circular wire. Single out a little piece of wire, dℓ, and label its current, I, and **r**, which points from the piece of wire to where we’re trying to find the field**.**  The field the tiny piece of wire sets up is:



So then integrating over the entire semi-circle.



Now find the field due to the long straight wire part. Just like above, single out a piece of wire, dℓ, and label I, r, and θ. And let ℓ be the distance between the beginning of the wire and the piece, dℓ.



Now the field created at the point by the piece of wire is:



Now integrate over the entire wire,



There are two wires, and so the total field is:

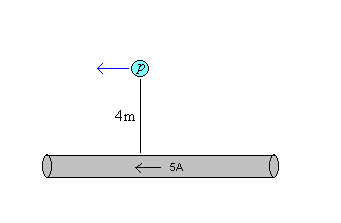


filling in the values,



**Problem**

Do long straight wire with p, and ask for **F**. Let υ = 107 m/s.



**Solution**

Do the magnitude/direction way first, and the cross product last. First observe the magnetic field at p is, as before,



Now the force on the proton is:

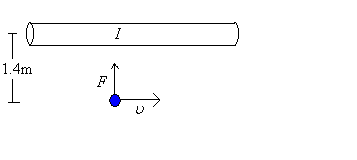


and so the force is pointing downward, clearly. What if we consider an e, instead of a p? Then,



\* In this case we see that the force is repulsive. This is because when using the right hand rule, **υ**×**B** gives the downward direction, but the negative sign of q reverses the direction to upward, just like as happened with **E**.

12. If an electron is traveling parallel to a long straight wire, at a distance of 1.4m, a velocity υ = 5.8×106m/s, and experiences a force F = 5.6×10-9N towards the wire, what is the magnitude and direction of the current in the wire?



First,

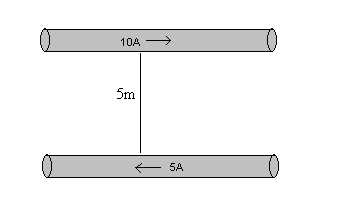


The field is due to a long straight wire, so:



**Problem**

What is force between the two wires in the example above? Assume that each are long, about 20m.



**Solution**

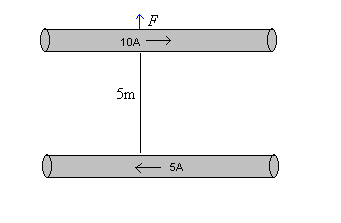
First we calculate the field produced by the lower wire at the top wire.



Then use the formula,

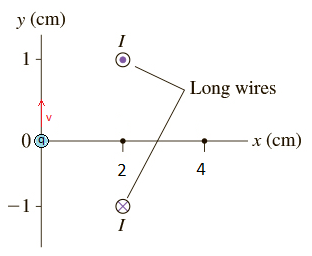


So the force is up.

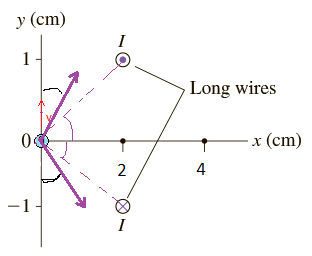


If we calculate the force of the top on the bottom, then we get 10-5N going down (forces are equal and opposite according to N3L). This illustrates a general rule. Currents going in opposite directions repel each other. While those going in the same direction attract.

**Question 7.** Two long straight wires carry a current 2A. What is the magnitude and direction of the force they exert on the charged particle q? You can take q = 9nC, and v = 3×105 m/s.



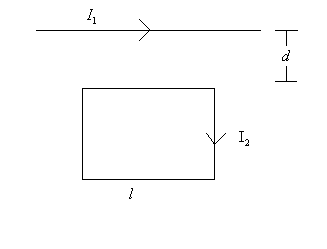
First we must find the magnetic field at the origin. The magnetic field vectors are shown below in purple. The angle (in black) they make with the vertical is given by the θ = tan-1(1/2) = 27° (same as the purple angle),



and so their component along the horizontal direction are both B = μ0I/2πR ∙ sin(27) = (4π×10-7)(2)/(2π√(0.01^2 + 0.02^2) ∙ sin(27) = 8.1μT. So the net field is **B** = 2(8.1μT) = 16.2μT pointing left. Then using the force law, the force on the charged particle is **F** = q**v**×**B** = (9nC)(3×105**j**)×(16.2μT**i**) = -43.7nN **k**, which points into the page.

**Problem**

Consider the two wires below. What is the net force the long straight wire exerts on the square current loop (with side length ℓ), and in what direction does it point? (Ignore the forces on the vertical sides of the square loop because they cancel out). Let I1 = 4A, I2 = 6A, d = 10cm, and ℓ = 5cm.



**Solution**

First we need the magnetic field through the two wires. The field at the top wire is:



and on bottom,



The force on the top wire is:



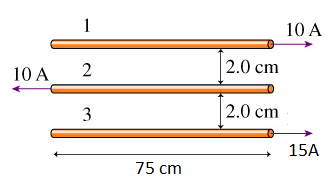
and on bottom,



So the net force is:



**Question 5.** Three parallel wires are shown. What is the force on the bottom wire?



The field on the bottom wire is:

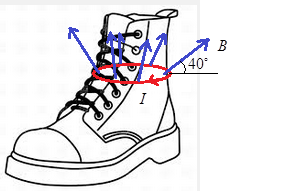


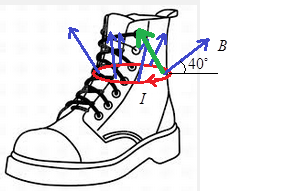
So then the force is:



**Question 2**

Magneto is testing out different levitation boots. The boot creates a tiny current, say I = 3mA, and radius R = 4cm circulating around itself. Then Magneto creates a magnetic field going through the boot/loop as shown making an angle of θ = 40° with the horizontal. What field strength will he have to create in order to levitate himself. Take his mass to be m = 80kg.





Force on a segment dℓ is shown in green and will be given by:



Only the vertical component of the force will not cancel. And so we’ll extract the vertical component:



And so then the force on the entire loop will be:

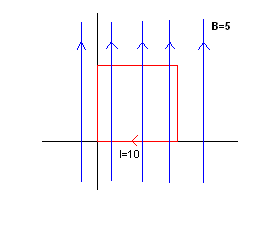


We need this force to be Fy = mg. So equating and solving for B we have:



**Problem**

Suppose we have a square (side length 2m) loop of current in the x-y plane, where the current is circulating around the loop clockwise with a magnitude of 10A. If we place it in a magnetic field of 5T pointing along the y-axis, which direction will it rotate?



**Solution**

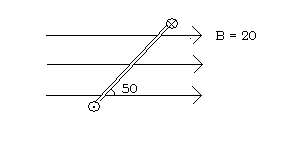
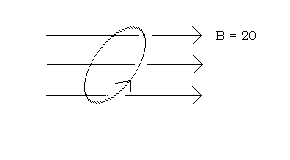
Well,



Therefore pointing our thumb in the direction of **x** our fingers curl in the direction CCW w/r to the positive x-axis. This is the direction of rotation.

**Problem**

A current loop (A = 2m2 and current 10mA) is embedded in a magnetic field. The plane of the current loop makes an angle of 50 degrees w/r to the field. What is the magnitude of the torque on the loop and in what direction (with respect to the picture on the right) will it turn – clockwise or counter-clockwise? The situation is given from two different perspectives. The second looks head on along the plane of the loop.



**Solution**

**μ** is:



and so,

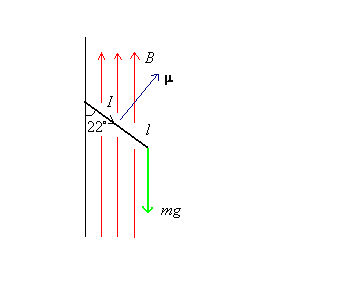
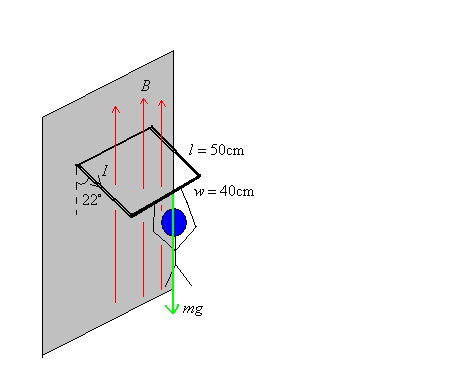


So,



**Problem**

Suppose we have N=1000 rectangular current loops (I = 0.2A in each) stacked on top of one another. If you glue one end to a wall, set up a vertical magnetic field, B, and hold on to the free end, how strong would B have to be in order to levitate you (m = 65kg) off the ground at an angle of 22 degrees?



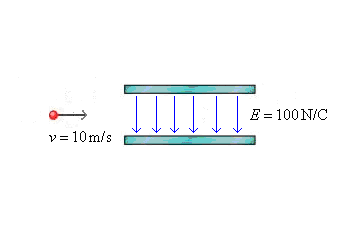
**Solution**

The situation is shown from two vantage points. We set the sum of the torques = 0 since the loop is in equilibrium. Therefore,



**Problem**

An electron heads towards an electric field. In order for the path of the electron to be undeflected through the plates, what magnetic field (strength and direction) must be set up between the plates?



**Solution**

In order for the path to be undeflected we need the magnetic force to cancel the electric force. So we need,

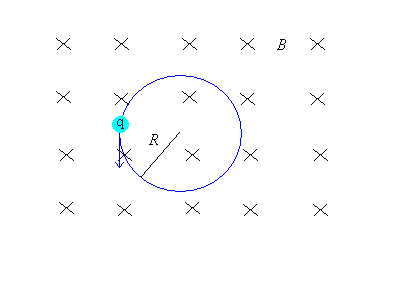


In order for the path to be undeflected, the force, FB = qυB, must point downwards, opposite the electric force. A magnetic field pointing into the page will accomplish this,



**Problem**

Suppose a charge, q, is sent through a magnetic field. It is observed to circulate CCW with a radius R = 15cm. If the mass of the particle is 4.6×10-27kg, and v = 5×106m/s, what is the sign and magnitude of the charge?



**Solution**

We have:

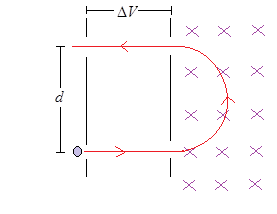


The direction of the force in the position shown for the charge must be to the right (**x**) in order for the circular motion to be supported. Given the direction of **υ**, and **B**, this requires q to be (+). So



**Example**

Suppose you accelerate a He nucleus with mass m = 6.67×10-27kg and charge q = 2e from rest through a potential difference ΔV = 12kV. What magnetic field strength, B, is required to make the charge traverse that semi-circular arc with diameter d = 10cm?

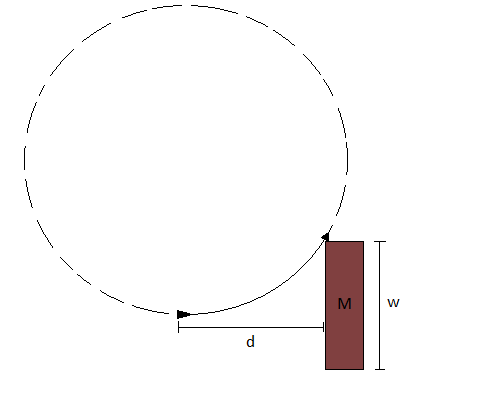


Speed of the charge will be given by: ΔPE = ΔKE → qΔV = (1/2)mv2 → v = √(2qΔV/m). Then the radius of its orbit will be: r = mv/qB. We want r = d/2 and so have d/2 = mv/qB → B = 2mv/dq. Filling in v we have:

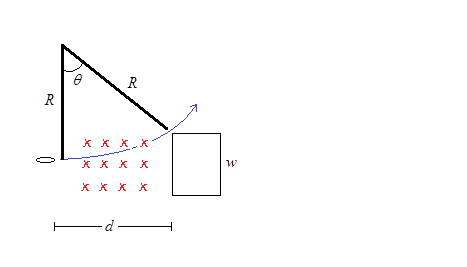


**Problem**

Suppose you’re Magneto and you create a magnetic field to deflect a bullet traveling with speed v = 500m/s, and charge q = 2pC. What field must you create if the bullet is aimed towards your middle (w = 75cm) and it is d = 50m away? Let mass of bullet be m = 10g.



So the path the bullet will take will be circular, as shown below:



We can write two expressions for R: one based on geometry and the other from physics. According to the picture we must have R – Rcosθ = w/2, whereas Rsinθ = d. This implies that:



Squaring both sides of the equations and adding we get:



Also,

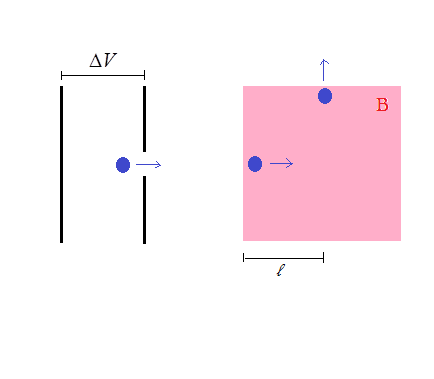


and so we have:



Filling in the numbers we get:

**Question 8**. A singly ionized Oxygen molecule (m = 5.31×10-26 kg) is accelerated through a potential difference 300kV, whereupon it enters a region of magnetic field, B. What field strength and direction is necessary to deflect the particle into a vertical trajectory in the given distance ℓ = 8cm? And which direction should the field be pointing in (left/right, up/down, into/out of the page)?



Field must point into page. And we have: R = mv/qB → B = mv/rq. Now r = ℓ = 8cm,

q = 1.6×10-19C, m is given and v comes from energy conservation: qΔV = (1/2)mv2 → v = √(2qΔV/m) = 1.35×106 m/s. And so B = 5.6T.

**Question 8**. EM waves with frequency f = 3.4THz are detected near a suspected black hole. If these waves are caused by electrons circling in a magnetic field, what is the black hole’s magnetic field strength? FYI, me = 9.11×10-31kg.

We have R = mv/qB. And the frequency is f = 1/T = 1/(d/v) = v/d = v/2πR. Filling in the radius, we have: f = v/[2πmv/qB) = qB/2πm. So B = 2πfm/q = 2π(3.4×1012)(9.11×10-31)/1.6×10-19 = 121 T.